

Hadron structure in manifestly Lorentz-invariant baryon chiral perturbation theory

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Abstract

We briefly outline the so-called extended on-mass-shell renormalization scheme for manifestly Lorentz-invariant baryon chiral perturbation theory which provides a simple and consistent power counting for renormalized diagrams. We comment on the role of chiral symmetry in the renormalization program and discuss as applications the mass and the electromagnetic form factors of the nucleon.

1 Introduction

Mesonic chiral perturbation theory (ChPT) [1, 2] has been tremendously successful and may be considered as a full-grown and mature area of low-energy particle physics (for a recent review, see, e.g., Ref. [3]). The prerequisite for an effective field theory program is (a) a knowledge of the most general effective Lagrangian and (b) an expansion scheme for observables in terms of a consistent power counting method [1]. In the mesonic sector the Lagrangian is known up to and including $\mathcal{O}(q^6)$ in the momentum and quark mass expansion [4]. The combination of dimensional regularization with the modified minimal subtraction scheme of ChPT [2] leads to a straightforward correspondence between the loop expansion and the chiral expansion in terms of momenta and quark masses at a fixed ratio, and thus provides a consistent power counting for renormalized quantities. In the extension to the one-nucleon sector [5] the correspondence between the loop expansion and the chiral expansion, at first sight, seems to be lost: higher-loop diagrams can contribute to terms as low as $\mathcal{O}(q^2)$ [5]. This problem has been eluded in the framework of the heavy-baryon formulation of ChPT [6], resulting in a power counting analogous to the mesonic sector. The price one pays consists of giving up manifest Lorentz invariance of the Lagrangian. In addition, at higher orders in the chiral expansion, the expressions due to $1/m$ corrections of the Lagrangian become increasingly complicated [7]. Finally, not all of the scattering amplitudes, evaluated perturbatively in the heavy-baryon framework, show the

correct analytical behavior in the low-energy region. In the following we will outline some recent developments in devising a renormalization scheme leading to a simple and consistent power counting for the renormalized diagrams of a manifestly Lorentz-invariant approach.

2 Manifestly Lorentz-Invariant Baryon Chiral Perturbation Theory and EOMS Scheme

In order to illustrate the issue of power counting, let us consider the lowest-order πN Lagrangian [5], expressed in terms of bare fields and parameters denoted by subscripts 0,

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}_0 \left(i\gamma_\mu \partial^\mu - m_0 - \frac{1}{2} \frac{\overset{\circ}{g}_{A0}}{F_0} \gamma_\mu \gamma_5 \tau^a \partial^\mu \pi_0^a \right) \Psi_0 + \dots, \quad (1)$$

where Ψ_0 and $\vec{\pi}_0$ denote a doublet and a triplet of bare nucleon and pion fields, respectively. After renormalization, m , $\overset{\circ}{g}_A$, and F refer to the chiral limit of the physical nucleon mass, the axial-vector coupling constant, and the pion-decay constant, respectively. The most general effective Lagrangian of the interaction of Goldstone bosons with nucleons consists of a string of terms

$$\mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \dots,$$

where the superscripts refer to the order in the derivative and quark-mass expansion [5]. In addition, one needs the most general effective Lagrangian of the mesonic sector [2, 4]

$$\mathcal{L}_\pi = \mathcal{L}_2 + \mathcal{L}_4 + \dots,$$

containing only even powers in the chiral expansion.

The aim is to devise a renormalization procedure generating, after renormalization, the following power counting: a loop integration in n dimensions counts as q^n , pion and fermion propagators count as q^{-2} and q^{-1} , respectively, vertices derived from \mathcal{L}_{2k} and $\mathcal{L}_{\pi N}^{(k)}$ count as q^{2k} and q^k , respectively. Here, q generically denotes a small expansion parameter such as, e.g., the pion mass. In total this yields for the power D of a diagram in the one-nucleon sector the standard formula

$$D = nN_L - 2I_\pi - I_N + \sum_{k=1}^{\infty} 2kN_{2k}^\pi + \sum_{k=1}^{\infty} kN_k^N, \quad (2)$$

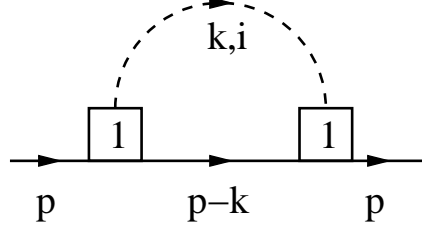


Figure 1: One-loop contribution to the nucleon self-energy. The number 1 in the interaction blobs refers to $\mathcal{L}_{\pi N}^{(1)}$.

where N_L , I_π , I_N , N_{2k}^π , and N_k^N denote the number of independent loop momenta, internal pion lines, internal nucleon lines, vertices originating from \mathcal{L}_{2k} , and vertices originating from $\mathcal{L}_{\pi N}^{(k)}$, respectively.

As an example, let us consider the one-loop contribution of Fig. 1 to the nucleon self-energy. According to Eq. (2), after renormalization, we would like to have the order

$$D = n \cdot 1 - 2 \cdot 1 - 1 + 1 \cdot 2 = n - 1. \quad (3)$$

Applying the $\widetilde{\text{MS}}$ renormalization scheme of ChPT [2, 5]—indicated by “r”—one obtains

$$\Sigma_{\text{loop}}^r = -\frac{3g_{Ar}^2}{4F_r^2} \left[-\frac{M^2}{16\pi^2} (\not{p} + m) + \dots \right] = \mathcal{O}(q^2),$$

where M^2 is the lowest-order expression for the squared pion mass. In other words, the $\widetilde{\text{MS}}$ -renormalized result does not produce the desired low-energy behavior of Eq. (3). This finding has widely been interpreted as the absence of a systematic power counting in the relativistic formulation of ChPT.

Recently, several methods have been suggested to obtain a consistent power counting in a manifestly Lorentz-invariant approach [8, 9, 10, 11, 12, 13, 14, 15]. Here, we will concentrate on the so-called extended on-mass-shell (EOMS) renormalization scheme [12]. The central idea of the EOMS scheme consists of performing additional subtractions beyond the $\widetilde{\text{MS}}$ scheme. Since the terms violating the power counting are analytic in small quantities, they can be absorbed by counterterm contributions. We will illustrate our approach in terms of the integral

$$H(p^2, m^2; n) = \int \frac{d^n k}{(2\pi)^n} \frac{i}{[(k-p)^2 - m^2 + i0^+][k^2 + i0^+]},$$

where $\Delta = (p^2 - m^2)/m^2 = \mathcal{O}(q)$ is a small quantity. We want the (renormalized) integral to be of the order $D = n - 1 - 2 = n - 3$. Applying the dimensional counting analysis of Ref. [16] (for an illustration, see the appendix of Ref. [17]), the result of the integration is of the form [12]

$$H \sim F(n, \Delta) + \Delta^{n-3}G(n, \Delta),$$

where F and G are hypergeometric functions and are analytic in Δ for any n . Hence, the part containing G for noninteger n is proportional to a noninteger power of Δ and satisfies the power counting. On the other hand F violates the power counting. The crucial observation is that the part proportional to F can be obtained by *first* expanding the integrand in small quantities and *then* performing the integration for each term [16]. This observation suggests the following procedure: expand the integrand in small quantities and subtract those (integrated) terms whose order is smaller than suggested by the power counting. In the present case, the subtraction term reads

$$H^{\text{subtr}} = \int \frac{d^n k}{(2\pi)^n} \frac{i}{[k^2 - 2p \cdot k + i0^+][k^2 + i0^+]} \Big|_{p^2=m^2}$$

and the renormalized integral is written as $H^R = H - H^{\text{subtr}} = \mathcal{O}(q^3)$ as $n \rightarrow 4$. In the infrared renormalization scheme of Becher and Leutwyler [9], one would keep the contribution proportional to G (with subtracted divergences when n approaches 4) and completely drop the F term.

3 The EOMS Scheme and Chiral Symmetry

In the chiral limit of massless u and d quarks, the QCD Lagrangian has a *global* $SU(2)_L \times SU(2)_R \times U(1)_V$ symmetry. As a consequence of this symmetry, Green functions involving the Noether currents are constrained by Ward identities. The Green functions may most efficiently be combined in a generating functional through a coupling of the quark bilinears to external fields [2]. In the framework of chiral perturbation theory, the generating functional is calculated by means of the most general effective mesonic and πN Lagrangians. By construction, the effective Lagrangian of the relativistic formulation is manifestly chirally invariant under *local* $SU(2)_L \times SU(2)_R \times U(1)_V$ transformations provided the external fields are transformed accordingly [2]. The *local* invariance of the Lagrangian guarantees that the chiral Ward identities of QCD (as well as their symmetry-breaking pattern) are encoded in the generating functional which is now determined through the *effective* field theory.

In the following we will briefly discuss the consequences of chiral symmetry for the renormalization program (for a similar discussion, see Ref. [9]). The tree graphs calculated in terms of the effective Lagrangian separately satisfy the Ward identities. Dimensional regularization is known to respect the symmetry relations induced by chiral symmetry for arbitrary n so that the corresponding regularized loop diagrams also satisfy the Ward identities. The one-loop diagrams may be divided into two parts: the first part is proportional to noninteger power(s) of the small expansion parameter(s) and the second part is analytic. The nonanalytic parts cannot be altered by changing the renormalization prescription and thus necessarily satisfy the Ward identities independently from the analytic parts. The analytic parts satisfy the Ward identities order by order in small expansion parameters. In our renormalization procedure we subtract all terms of the expansion of the analytic parts of the one-loop diagrams that violate the power counting. These subtraction terms satisfy the Ward identities order by order. Hence the renormalized diagrams also respect the Ward identities.

For multi-loop diagrams the procedure is analogous albeit technically more complicated. For example, two-loop diagrams may contain parts which are nonanalytic in the expansion parameter(s) and which cannot be altered by the renormalization condition. These parts do not violate the power counting and satisfy the Ward identities separately. However, there may also be contributions which are nonanalytic but depend on the renormalization condition for the one-loop sub-diagrams. If the finite parts of the counterterms are fixed so that the power counting is satisfied at the one-loop level, then these parts of two-loop diagrams, combined with contributions from counterterm diagrams renormalizing one-loop sub-diagrams, satisfy power counting (see Ref. [17]). As long as the renormalization of the one-loop diagrams respects chiral symmetry, the above second type of nonanalytic parts also satisfies the Ward identities. Finally, the third part is analytic and can be altered by counterterms. Starting from here, the argument is as for the one-loop case. This (standard) procedure of renormalization is then performed iteratively for diagrams with an increasing number of loops.

4 Applications

Let us first discuss the result for the mass of the nucleon at $\mathcal{O}(q^4)$ [12],

$$m_N = m + k_1 M^2 + k_2 M^3 + k_3 M^4 \ln \left(\frac{M}{m} \right) + k_4 M^4 + \mathcal{O}(M^5), \quad (4)$$

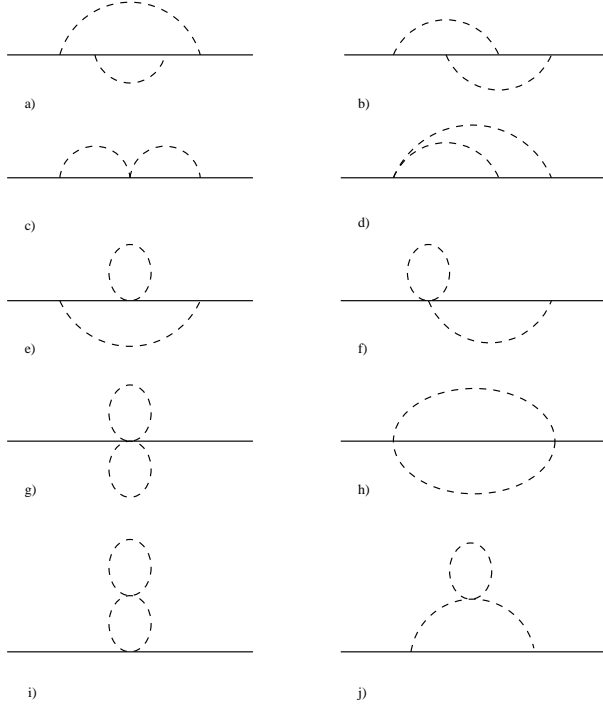


Figure 2: Two-loop topologies of the nucleon self-energy. Crossed diagrams are not shown.

where the coefficients k_i are given by

$$\begin{aligned}
 k_1 &= -4c_1, \quad k_2 = -\frac{3\overset{\circ}{g}_A^2}{32\pi^2 F^2}, \quad k_3 = \frac{3}{32\pi^2 F^2} \left(8c_1 - c_2 - 4c_3 - \frac{\overset{\circ}{g}_A^2}{m} \right), \\
 k_4 &= \frac{3\overset{\circ}{g}_A^2}{32\pi^2 F^2 m} (1 + 4c_1 m) + \frac{3}{128\pi^2 F^2} c_2 + \frac{1}{2} \alpha.
 \end{aligned} \tag{5}$$

Here, $\alpha = -4(8e_{38} + e_{115} + e_{116})$ is a linear combination of $\mathcal{O}(q^4)$ coefficients [7]. In order to obtain an estimate for the various contributions of Eq. (4) to the nucleon mass, we make use of the set of parameters c_i of Ref. [18],

$$c_1 = -0.9 m_N^{-1}, \quad c_2 = 2.5 m_N^{-1}, \quad c_3 = -4.2 m_N^{-1}, \quad c_4 = 2.3 m_N^{-1} \tag{6}$$

which were obtained from a (tree-level) fit to πN scattering threshold parameters. Using the numerical values

$$g_A = 1.267, \quad F_\pi = 92.4 \text{ MeV}, \quad m_N = 938.3 \text{ MeV}, \quad M_\pi = 139.6 \text{ MeV},$$

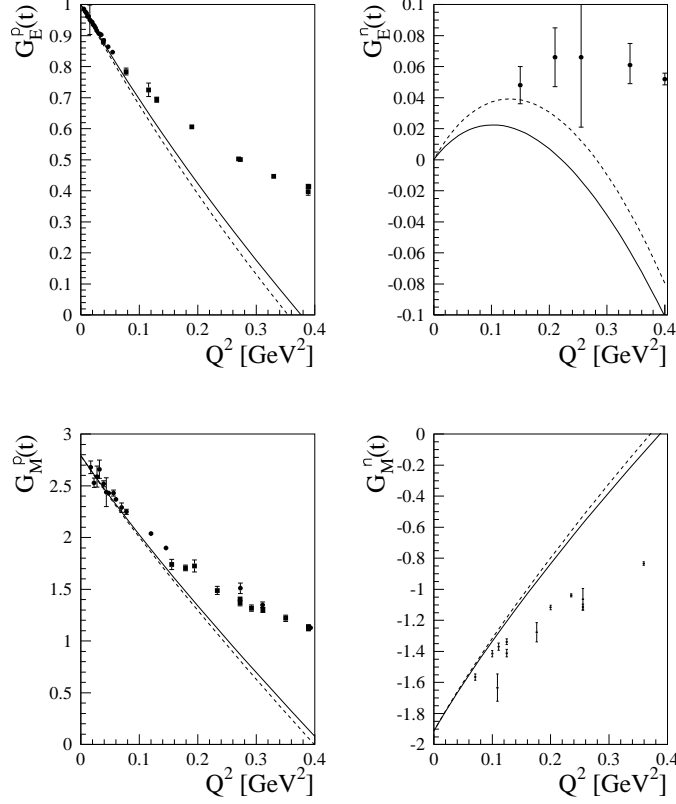


Figure 3: The Sachs form factors of the nucleon at $\mathcal{O}(q^4)$. The solid and dashed lines refer to the results in the EOMS scheme [20] and the infrared regularization [21], respectively.

we obtain for the mass of nucleon in the chiral limit (at fixed $m_s \neq 0$):

$$m = m_N - \Delta m = [938.3 - 74.8 + 15.3 + 4.7 + 1.6 - 2.3] \text{ MeV} = 882.8 \text{ MeV}$$

with $\Delta m = 55.5 \text{ MeV}$. Here, we have made use of an estimate for α obtained from the σ term (see Ref. [14] for details). The chiral expansion reveals a good convergence and it will be interesting to further study the convergence at the two-loop level [19] (see Fig. 2).

As another example, let us consider the electromagnetic form factors of the nucleon which are defined via the matrix element of the electromagnetic current operator as

$$\langle N(p_f) | J^\mu(0) | N(p_i) \rangle = \bar{u}(p_f) \left[\gamma^\mu F_1^N(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} F_2^N(Q^2) \right] u(p_i), \quad N = p, n,$$

where $q = p_f - p_i$ is the momentum transfer and $Q^2 \equiv -q^2 = -t \geq 0$. Figure 3 shows the results for the electric and magnetic Sachs form factors $G_E = F_1 - Q^2/(4m_N^2)F_2$ and $G_M = F_1 + F_2$ at $\mathcal{O}(q^4)$ in the EOMS scheme (solid lines) [20] and the infrared renormalization (dashed lines) [21]. The $\mathcal{O}(q^4)$ results only provide a decent description up to $Q^2 = 0.1 \text{ GeV}^2$ and do not generate sufficient curvature for larger values of Q^2 . We conclude that the perturbation series converges, at best, slowly and that higher-order contributions must play an important role. It remains to be seen to what extent a *consistent* inclusion of vector mesons [14] improves the quality of the description.

5 Summary and Outlook

The EOMS scheme allows for a simple and consistent power counting in manifestly Lorentz-invariant baryon chiral perturbation theory [12]. Since it can also be applied at the multi-loop level [13, 17] it will be interesting to consider selected two-loop examples and address the question of convergence. Moreover, the infrared renormalization of Becher and Leutwyler has been reformulated in a form analogous to the EOMS renormalization scheme [22] and can thus also be applied to multi-loop diagrams with an arbitrary number of particles with arbitrary masses (see also Ref. [23]). Clearly, the method has a large potential and it will be interesting to apply it to electromagnetic processes such as Compton scattering and pion production.

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